

Performance Evaluation of LDPC Codes Over Various Channels

Saba Farheen N.S.¹, Asiya Hazreena², Jose Alex Mathew³, Ghouse Ahamed⁴

^{1,2,3}Dept. of E&CE, P.A. College of Engineering, Mangalore, Karnataka

⁴Dept. of E&CE, H.M.S Institute of Technology, Tumkur, Karnataka

¹Saba083@gmail.com, ²hazareenaut@yahoo.co.in,

³aymanamkuzhy@gmail.com, ⁴ghouse.ahamed@gmail.com

Abstract: Due to the rapid evolution of global wireless communication which demands for high data rate transmission via satellites, which in turn requires spectrally efficient modulation technique and power efficient forward-error correction (FEC), schemes. The main objective of any communication system is error free transmission with maximum possible data rate. Noisy communication channels are the major problems in this case. To overcome this problem one can use the channel coding along with the suitable modulation scheme. Thus the objective of Channel coding and modulation is to produce an appropriate signal waveform to cope with the noisy channel. Low density parity check (LDPC) codes are one of the best error correcting codes in today's coding world and are known to approach the Shannon limit. As with all other channel coding schemes, LDPC codes add redundancy to the uncoded input data to make it more immune to channel impairments. The impact of low-Density Parity-Check code (LDPC) on the performance of system under Binary Phase Shift Keying (BPSK), Quadrature Phase Shift Keying (QPSK) and Orthogonal Frequency Division multiplexing (OFDM) over an Additive White Gaussian Noise (AWGN) and other fading (Rayleigh, Rician and Nakagami) channels will be investigated in this work. Obtained results will show that LDPC can improve transceiver system for various channel types.

Keywords: LDPC; BPSK; QPSK; OFDM; Rayleigh; Rician; Nakagami

1. INTRODUCTION

LDPC codes were developed by Robert Gallager in his PhD thesis at MIT in 1962 [1]. These codes were ignored for about 30 years and rediscovered in the late 1990s by D. J. C. MacKay and R. M. Neal [2]. In 2001, T.J Richardson, A. Shokrollahi, and R. Urbanke proved that the performance of LDPC codes is close to the Shannon limit [3]. LDPC codes have certain advantages over other codes, e.g. turbo codes. They not only have a simple description of their code structure but can also have a fully parallelizable decoding implementation [3]. Because of their excellent forward error correction properties, LDPC codes are set to be used as a standard in Digital Video Broadcasting (DVB-S2) and 4G mobile communication [4]. LDPC codes have certain advantages over other codes, e.g. turbo codes. They not only

have a simple description of their code structure but can also have a fully parallelizable decoding implementation [5]. Also, their minimum distance ($d_{\min}$) increases proportionally with an increase in the block length [6].

LDPC codes have attracted a lot of attention in recent years. The codes have several properties, which make them favorable choices for real-time high-throughput communications. First, the codes are capacity-approaching. Second, the codes can be efficiently decoded by parallel iterative decoding algorithms with low latency [7]. LDPC was and still getting the concern of researchers to evaluate and develop its performances for many applications.

In addition to using channel coding for better error performance, the technique used for modulating the coded signal is also very important as it transforms the signal waveforms and enables them to better withstand channel distortions.

When the modulated signal travels through the channel, it gets distorted by noise and fading. The noise is generally modeled as AWGN as it is easier to treat noise as additive rather than multiplicative. A variety of models for fading have been proposed by researchers over the years, out of which Rayleigh, Rician and Nakagami have become very popular [8-10]. Rayleigh fading is used for modeling severe fading conditions and Rician fading is used for modeling fading conditions where a LOS (line-of-sight) exists between transmitter and receiver, that is, where fading conditions are less severe than Rayleigh fading. The Nakagami fading provides a very good fit for all fading conditions ranging from very severe to no fading because of the presence of an adaptive fading parameter 'm' called shape factor [11-12].

For mobile phones, LDPC codes may prove a better choice, since they can employ a fully parallelizable decoder. It has also been observed that LDPC codes outperform turbo codes on the Rayleigh fading channel. In this work, the LDPC has been applied with assistance of BPSK/QPSK and OFDM modem scheme for transmission over AWGN, Rician,

Rayleigh and Nakagami fading channels, which requires no bandwidth expansion. The evaluation of Bit Error Rate (BER) performance of the LDPC coded-BPSK, QPSK and OFDM is achieved over four types of channel.

2. CODE STRUCTURE

A regular (j, L) LDPC code is defined by an $(n-k) \times n$ parity-check matrix with n block length of the code and k information bits generated by the binary source. Such matrix having exactly j ones in each column and exactly L ones in each row, where $j < L$ and both are small compared to n . An irregular LDPC matrix is also sparse, but not all rows and columns contain the same number of ones. Fig. 2 shows the parity-check matrix of a $(3, 6)$ LDPC code [14].

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Fig.1 A regular $(3, 6)$ parity-check matrix H

By the definition of regular LDPC codes, every parity-check equation involves exactly L bits, and every bit is involved in exactly j parity-check equations. Observe that the fraction of ones in a regular (j, L) LDPC matrix is L/n . The “low density” terminology derives from the fact that this fraction approaches zero as n tends to ∞ [15]. In contrast, the average fraction of ones in a purely random binary matrix (with independent components equally likely to be zero or one) is $1/2$.

Recall that an $(n-k) \times n$ parity-check matrix H defines a code in which then bits of each codeword satisfy a set of $(n-k)$ parity-check equations. The Tanner graph contains n “variable” nodes, one for each codeword bit, and $(n-k)$ “check” nodes, one for each of the parity-check equations [16]. Fig. 3 shows the Tanner graph corresponding to the H matrix.

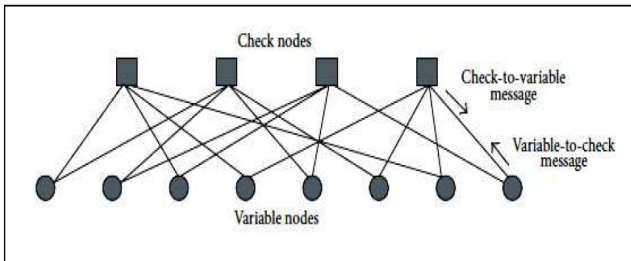


Fig. 2. Tanner graph representation of an LDPC code

An LDPC code parity-check matrix is called (w_c, w_r) -regular if each code bit is contained in a fixed number, w_c , of parity

checks and each parity-check equation contains a fixed number, w_r , of code bits. A regular LDPC code will have:

$$m \cdot W_r = n \quad (1)$$

Where w_c and w_r are number of ones in each column and row for regular parity check matrix of LDPC code respectively, m vertices for the paritycheck equations.

For an irregular parity-check matrix it must designate the fraction of columns of weight i by v_i and the fraction of rows of weight i by h_i . Collectively the set v and h is called the degree distribution of the code ones in its parity-check matrix. Similarly, for an irregular code [16]:

$$m(\sum_i h_i \cdot i) = n(\sum_i v_i \cdot i) \quad (2)$$

System Model

The system model used is shown in Fig. 3. The data to be transmitted over the channel was randomly generated by the binary source. The binary source is assumed to be memoryless, which is often the result of source coding, and therefore all information sequences are equally probable. This data is coded by using LDPC codes. After the coded bit sequence has been obtained, it is applied to a modulator. This modulated waveform is transmitted over the channel in the presence of AWGN. The received signal is passed through demodulator and decoder where the errors are detected and corrected. The various blocks used in the model have been described in detail below.

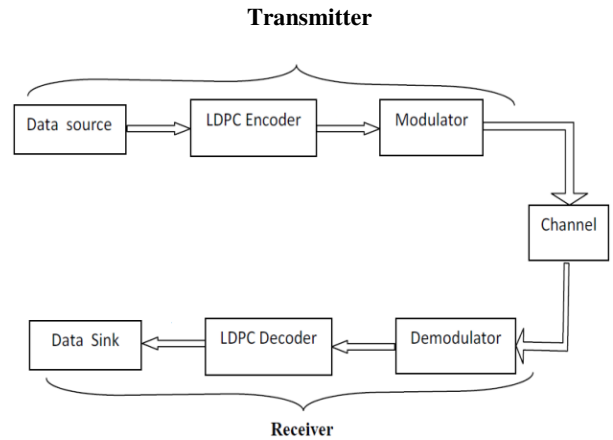


Fig 3 System model

3. ENCODING

An encoding algorithm for a binary linear code of dimension k and block length n is an algorithm that computes a codeword from k original bits $x_1, \dots, x_k$. To compare algorithms against each other, it is important to introduce the concept of cost, or operations. For the purposes of this note the cost of an algorithm is the number of arithmetic operations over F_2 that the algorithm uses.

To perform the encoding of these codes one needs to first convert the parity-check matrix into systematic form or equivalently into an upper triangular form using either of the mathematical operations like Gauss-Jordan\ elimination\ seidel.

The generator matrix for a code with parity-check matrix H obtained by performing either of the mathematical operations on H is in the form

$$H=[A, I_{n-k}] \quad (3)$$

Where, A is a $(n - k) \times k$ binary matrix and I_{n-k} is the size $n-k$ identity matrix.

The generator matrix is then

$$G=[I_k, A^T] \quad (4)$$

The resultant matrix G is then used for encoding which in-general is a dense matrix i.e, the sparse nature of the parity check matrix is lost due to the row operations, as result the complexity of encoder grows to n^3 [1].

To fix this n^3 complexity one need to follow an efficient encoding technique which involves efficient row and column swaps to convert the parity check matrix into systematic form., as a result of this the sparse nature of the parity check matrix is preserved and thus the complexity of the encoder is reduced to n [1].

4. DECODING

One of the most widely used decoding methods for LDPC codes is based on belief propagation. It is performed by applying the maximum a posteriori (MAP) algorithm. This algorithm aims at minimizing the bit error rate of the decoded sequence and iteratively calculates the a posteriori probabilities [9].

The MAP algorithm computes the a posteriori probability of each state transition given the noisy observation at the receiver. There is a one to one correspondence between a state transition and its corresponding code symbol. The states connected by the MAP-found state transition need not form a continuous path. The algorithm computes the a posteriori probabilities (APP) of each possible state transition and chooses the one which is more likely (highest probability) [8].

Consider a regular (j, k) LDPC code with v as the Log-Likelihood Ratio (LLR) message passed from a variable node of degree j to a check node of degree k , given as [3],

$$v = v_o + \sum_{i=1}^{j-1} n \quad (5)$$

In (3), v_o is intrinsic information conditioned on the channel output, and r_i for all, is the extrinsic information. Extrinsic information is part of the overall LLR stemming from the observation of the received samples. The check nodes update rule is obtained by noticing the duality between variable and check nodes. It is based upon the well known tanh rule and it is given as

$$\frac{\tanh r}{2} = \prod_{i=1}^{k-1} \frac{\tanh v_i}{2} \quad (6)$$

Where v_i , for all $i=1, \dots, k-1$, are the incoming LLRs from the neighboring edges.

5. RESULTS AND DISCUSSION

In this paper, the performance of LDPC codes are evaluated over four types of channel (AWGN, Rayleigh, Rician and Nakagami), with a code rate (R) = 1/2 for each channel. Jakes Doppler filter impulse response of fading channels is employed for all simulations. A (32400, 64800) regular LDPC coded bit stream was used. The simulations are applied on system model shown in Figure (1), using the MATLAB software package. The results are displayed as graphs in which the (BER) is plotted versus (SNR), measured in decibel (dB).

From the results of simulation it will be clear that this code achieved significant improvement for SNR at low BER. For wireless communication (Rayleigh, Rician and Nakagami channels) 6.5 to 9 dB code gain can be achieved for 1/2 code rate with low range of SNR at BER of 10^{-4} .

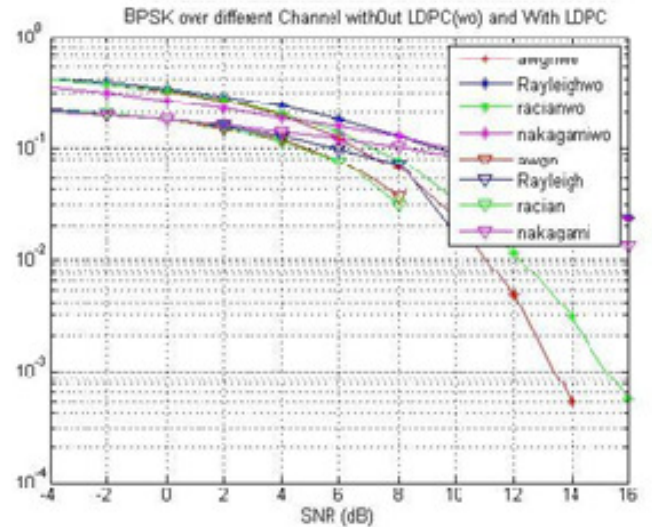


Fig 4: Comparison of uncoded BPSK with LDPC coded BPSK

Fig. 4 shows that LDPC coded BPSK modulated signal which gives a higher coding gain over the use of un-coded BPSK. So the performance of coded BPSK is much better than the performance of uncoded BPSK.

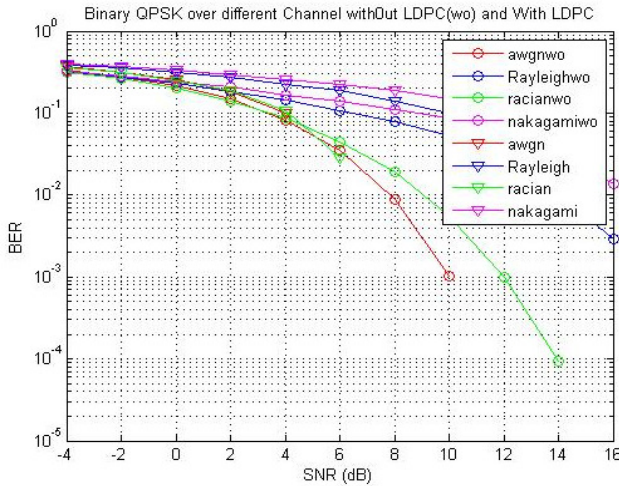


Fig 5: Comparison of uncoded QPSK with LDPC coded QPSK

Fig. 5 shows that LDPC coded BPSK modulated signal which gives a higher coding gain over the use of un-coded QPSK. So the performance of coded BPSK is much better than the performance of uncoded QPSK.

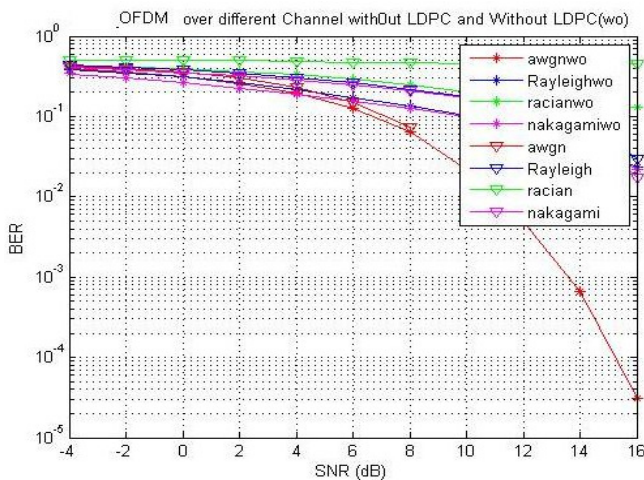


Fig 6: Comparison of uncoded OFDM with LDPC coded OFDM.

Fig. 6 shows that LDPC coded OFDM modulated signal which gives a higher coding gain over the use of un-coded OFDM. As a result the performance of coded OFDM is much better than the performance of uncoded OFDM.

6. CONCLUSION

LDPC codes have been studied a lot in the last years and huge progresses have been made in the understanding and ability to design iterative coding systems. The iterative decoding approach is already used in turbo codes but the structure of LDPC codes give even better results. In many cases they allow a higher code rate and also a lower error floor rate. In other word to achieve good coding gain performance, good LDPC code design is essential.

It has been observed that the system with LDPC codes has good performance in Rician, Rayleigh and Nakagami fading channel. More than 8 dB code gain over un-coded system can be achieved with 1/2 code rate of LDPC codes over these channels. Also this code rate helps in achieving beneficial gain while maintaining spectrum efficiency because of no more redundant information added to the message.

REFERENCES

- [1] Gallager, R. G., "Low Density Parity Check Codes", Number 21 in Research monograph series. MIT Press, Cambridge, Mass, 1963.
- [2] MacKay, D. J. C. and R. M. Neal, "Near Shannon limit performance of low density parity check codes," *Electronics Letters*, vol. 32, no. 18, pp. 1645-1646, Aug. 1996.
- [3] T. J. Richardson, A. Shokrollahi, and R. Urbanke, "Design of capacity approaching irregular low-density parity-check codes," *IEEE Trans. Inform. Theory*, vol.47, no. 2, pp. 619-637, Feb. 2001
- [4] S.Y. Chung, G. David Forney, T.J. Richardson, and R. Urbanke "On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit" *IEEE Communications Letters*, vol. 47, no. 2, pp. 58-60, February 2001.
- [5] Hou J. P. H. Siegel. and L. B. Milstein, "Performance analysis and code optimization of low density parity-check codes on rayleigh fading channels," in *IEEE Journal on Selected Areas in Communications*, no. 15, May 2001, pp. 924-934.
- [6] Eliss Simranjit Singh. Surbhi Sharma. Rajesh Khanna" Performance of Uncoded and LDPC Coded MSK and GMSK in NakagamiFading", *International Journal of Recent Trends in Engineering*, Vol2, No.6, November 2009.
- [7] Xudong Ma, Onconstructing Low Density parity -CheckCodes , PhD Thesis , University of Waterloo, Ontario, Canada , 2007.
- [8] Lou I. ILUNGA, "Adaptive, Lou I. ILUNGA, "Adaptive Turbo-coded OFDM", MSc thesis, Virginia Polytechnic
- [9] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Information Theory*, no. 2, pp. 284-287, 1974
- [10] M. Nakagami, "The m-distribution. A general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, W.C. Hoffman, Ed. Elmsford, NY, Pergamon Press, 1960.
- [11] Li Tang, Zhu Hongbo, "Analysis and simulation of Nakagami fading channel with matlab" *Asia-Pacific Conference on Environmental Electromagnetics CEEM 2003*, pp. 490-494, Nov. 2003.
- [12] Gayarti S. Prabhu, P.Mohana Shankar "Simulation of flat fading using MATLAB for classroom instruction", *IEEE Transactions on Education* Vol.45. No.1, pp. 19-25, Feb. 2002.
- [13] R. M. Tanner, "A recursive approach to low complexity codes", *IEEE Trans. Inform. Theory*, vol. 27, no. 5, pp.533-547, September 1981.
- [14] S. Lin and D. Costello, "Error Control Coding", Prentice Hall, 2nd edition, 2004.
- [15] J. Barry, "Digital Communication", Springer, 3rd edition, 2003.
- [16] Shaikh Faisal Zaheer, "Improved Rate-Compatible Low-Density Msc thesis, King Fahd University, Saudi Arabia, 2006.